MAP 2302
Quiz – Spring 2020
7.1 & 7.2 – Equation solving & Definitions

Name:
Date:

~~KEY~~

6 pts

① Evaluate the Laplace transform $\mathcal{L}\{e^{-3t}\}$ using the definition. Show your work (step by step) to receive full credit.

$$\mathcal{L}\{e^{-3t}\} = \int_0^\infty e^{-st}(e^{-3t})\,dt = \lim_{b\to\infty}\int_0^b e^{-(s+3)t}\,dt \qquad \text{let } s > -3$$

$$= \lim_{b\to\infty}\left[\frac{1}{-(s+3)}e^{-(s+3)t}\Big|_{t=0}^{t=b}\right.$$

$$= \lim_{b\to\infty}\left[\frac{-1}{s+3}e^{-(s+3)(0)} + \frac{1}{s+3}e^{-(s+3)b}\right] = \boxed{\frac{1}{s+3}}$$

(the $e^{-(s+3)b}$ term $\to 0$)

② Use Laplace transforms to solve the initial value problem $y' + 6y = e^{4t}$ $y(0) = 2$

6 pts

$$\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{e^{4t}\}$$

$$s\mathcal{L}\{y\} - \underbrace{y(0)}_{2} + 6\mathcal{L}\{y\} = \frac{1}{s-4}$$

$$(s+6)\mathcal{L}\{y\} = \frac{1}{s-4} + 2$$

$$\mathcal{L}\{y\} = \frac{1}{(s+6)(s-4)} + \frac{2}{s+6}$$

$$y = \mathcal{L}^{-1}\left\{\frac{1}{(s+6)(s-4)}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s+6}\right\}$$

$$y = \mathcal{L}^{-1}\left\{\frac{A}{s+6} + \frac{B}{s-4}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s+6}\right\}$$

$$y = Ae^{-6t} + Be^{4t} + 2e^{4t}$$

$$A(s-4) + B(s+6) = 1$$

$$A = \frac{-1}{10} \qquad B = \frac{1}{10}$$

$$\boxed{y = \frac{19}{10}e^{-6t} + \frac{1}{10}e^{4t}}$$

(3) Use Laplace transforms to solve the initial value problem $y'' + 5y' + 4y = 0$ $\quad y'(0) = 0$, $y(0) = 1$ — 6pts

$$\mathcal{L}\{y''\} + 5\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{0\}$$

$$s^2\mathcal{L}\{y\} - s\cdot y(0) - y'(0) + 5[s\mathcal{L}\{y\} - y(0)] + 4\mathcal{L}\{y\} = 0$$

($y(0) = 1$, $y'(0) = 0$, $y(0) = 1$)

$$s^2\mathcal{L}\{y\} - s + 5\mathcal{L}\{y\} - 5 + 4\mathcal{L}\{y\} = 0$$

$$\mathcal{L}\{y\}(s^2 + 5s + 4) = s + 5$$

$$\mathcal{L}\{y\} = \frac{s+5}{s^2+5s+4} = \frac{s+5}{(s+4)(s+1)} = \frac{A}{s+4} + \frac{B}{s+1}$$

$$y = \mathcal{L}^{-1}\left\{\frac{A}{s+4} + \frac{B}{s+1}\right\} = \boxed{Ae^{-4t} + Be^{-t}}$$

$$A(s+1) + B(s+4) = s$$
$$(A+B)s + A + 4B = s$$

$A + B = 1 \qquad A + 4B = 5$

$$A + B = 1$$
$$-(A + 4B = 5)$$
$$-3B = -4$$
$$\boxed{B = \frac{4}{3}}$$

$$A + B = 1$$
$$A + \frac{4}{3} = 1$$
$$\boxed{A = -\frac{1}{3}}$$

$$\boxed{y = -\frac{1}{3}e^{-4t} + \frac{4}{3}e^{-t}}$$